Distribution Natural Waves on the Viscoelastic Cylindrical Body in Plane Strain State

Author's Details:
[(1)]**Safarov Ismail Ibrohimovich** -Doctor of physical and mathematical sciences, professor, Bukhara Technological Institute of Engineering, Republic of Uzbekistan. [(2)]**Teshaev Mukhsin Khudoyberdiyevich**- Candidate of Physical Mathematical Sciences, docent, Bukhara Technological Institute of Engineering, Republic of Uzbekistan.15 K. Murtazoyev Street, Bukhara site [(3)]**Boltayev Zafar Ixtiyarovich –** researcher, Bukhara Technological Institute of Engineering, Republic of Uzbekistan.15 K. Murtazoyev Street [(4)]**Akhmedov Maqsud Sharipovich –** senior researcher, Bukhara Technological- Institute of Engineering Republic of Uzbekistan. E-mail:, 15 K. Murtazoyev Street. [(5)]**Rajabov Ozod Isroilovich-** researcher, Bukhara Technological Institute of Engineering, Republic of Uzbekistan. 15 K. Murtazoyev Street

Abstract: *In this paper the distribution of natural waves in the viscoelastic disc is discussed. The spectral problem is reduced to solving a system of ordinary differential equations of the first order with variable complex coefficients. The solution of differential equations is expressed by a special cylindrical Bessel functions and Hankel. The frequency equations are solved by numerical methods Muller and Gauss. The problem is solved numerically by the Godunov orthogonal sweep method and Mueller. Compares the numerical results.*
keyword*: natural oscillations, dissipative properties, the environment, spectral problem, the natural frequency, the wave number.*

1. Introduction.

One of the central tasks of dynamic elasticity theory is the study of the spread of a perturbation of the stress-strain state in deformed bodies (including viscoelastic properties) with geometric structures [1,2,3,4]. The main features are the length of the waveguide in one direction, as well as restrictions and localization of the wave beam in other directions. Accounting for the damping capacity of the waveguide material plays an important role in the dynamic behavior of the design.

In an infinite homogeneous isotropic medium there are only waves P and S. However, where there is a surface separating media with different elastic properties can propagate waves. The amplitudes of these waves decreases with the distance from the surface.

The study of the properties of guided modes is also important in connection with the development of techniques for the use of acoustic emission intensity level assessment of structural elements [5,6,7]. The electronic technology has been widely used broad beams of surface waves. The most important for the practice of (seismic) types of surface waves are Rayleigh waves propagating along the free surface of the solid medium. The dynamic theory of elasticity is known [8,9] that the surface Rayleigh wave covers half with straight boundaries. The study wave propagation in the body with curved areas is an urgent task.

2. Statement of the problem and methods of solution.

Let us consider the propagation of surface waves on the cylindrical body located in a flat deformable state. The equation of a viscoelastic body motion in cylindrical coordinates (r, θ), takes the form [10,11]

$$\rho\frac{\partial^2 u_r}{\partial t^2}=(\tilde{\lambda}+2\tilde{\mu})\frac{\partial\Delta}{\partial r}-\frac{2\tilde{\mu}}{r}\frac{\partial\omega_z}{\partial\theta}$$
$$\rho\frac{\partial^2 u_\theta}{\partial t^2}=(\tilde{\lambda}+2\tilde{\mu})\frac{1}{r}\frac{\partial\Delta}{\partial\theta}+2\tilde{\mu}\frac{\partial\omega_z}{\partial r} \qquad (1)$$

where

$$\Delta=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1\partial}{r}\frac{u_\theta}{\partial\theta};\quad \omega_z=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}-\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right), \qquad (2)$$

$$\tilde{\lambda}\, f(t)=\lambda_0\left[f(t)-\int_0^t R_\lambda(t-\tau)f(\tau)d\tau\right];$$
$$\tilde{\mu}\, f(t)=\mu_0\left[f(t)-\int_0^t R_\mu(t-\tau)f(\tau)d\tau\right], \qquad (3)$$

$f(t)$– arbitrary function of time; $R_\lambda(t-\tau)$ and $R_\mu(t-\tau)$– relaxation kernel; λ_0 and μ_0– instantaneous elastic moduli; ρ - material density, $\tilde{\nu}=const=\nu_0$ - Poisson's ratio.

External loads on the free cylindrical surface $r=R_1$ available, i.e. $\sigma_{rr}=0,\ \tau_{r\theta}=0$ or

$$\left(\sigma_{rr}\right)_{r=R} = \frac{\tilde{E}}{1+\tilde{\nu}}\left[\frac{1}{1+\tilde{\nu}}\Delta+\frac{\partial u_r}{\partial r}\right]\Bigg|_{r=R} = 0 \quad , \quad \left(\tau_{r\theta}\right)_{r=R} = 2\tilde{\mu}\left[\omega_z+\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta}-u_\theta\right)\right]\Bigg|_{r=R} = 0 \;. \quad (4)$$

$\tilde{E}$ – modulus operator, which has the form [12,13]:

$$\tilde{E}f(t)=E_{01}\left[f(t)-\int_0^t R_E(t-\tau)f(t)d\tau\right] (5)$$

$R_E(t-\tau)$– relaxation kernel; E_{01}– instantaneous modulus.

In the center of the cylindrical body (r = 0) is assumed, that the movement is limited. Replace the relations (3) and (5) approximate species [12]

$$\bar{\mu} = \mu_0\left[1-\Gamma_\mu^C(\omega_R)-i\Gamma_\mu^S(\omega_R)\right],\ \bar{\lambda}=\lambda_0\left[1-\Gamma_\lambda^C(\omega_R)-i\Gamma_\lambda^S(\omega_R)\right],\ \ \bar{E}=E_{01}\left[1-\Gamma_E^C(\omega_R)-i\Gamma_E^S(\omega_R)\right],\ (4)$$

where ω_R - real constant,

$$\Gamma_E{}^C(\omega_R)=\int_0^\infty R_E(\tau)\cos\omega_R\tau\, d\tau\, ,\Gamma_E{}^S(\omega_R)=\int_0^\infty R_E(\tau)\sin\omega_R\tau\, d\tau\, ,$$

$$\Gamma_\mu{}^C(\omega_R)=\int_0^\infty R_\mu(\tau)\cos\omega_R\tau\, d\tau\, ,\Gamma_\mu{}^S(\omega_R)=\int_0^\infty R_\mu(\tau)\sin\omega_R\tau\, d\tau\, ,$$

$$\Gamma_\lambda{}^C(\omega_R)=\int_0^\infty R_\lambda(\tau)\cos\omega_R\tau\, d\tau\, ,\ \Gamma_\lambda{}^S(\omega_R)=\int_0^\infty R_\lambda(\tau)\sin\omega_R\tau\, d\tau$$

- respectively, the cosine and sine Fourier transform of the relaxation of the core material. After some simple transformations ($R_\lambda = R_\mu = R_E = R_1$) equations motion (1) can be converted into a

$$\begin{aligned}\frac{\partial^2 u_r}{\partial t^2} &= c_1^2 T_1\frac{\partial \Delta}{\partial r}-\frac{2}{r}c_2^2 T_1\frac{\partial \omega_z}{\partial \theta};\\ \frac{\partial^2 u_\theta}{\partial t^2} &= c_1^2 T_1\frac{1}{r}\frac{\partial \Delta}{\partial \theta}+2c_2^2 T_1\frac{\partial \omega_z}{\partial r},\end{aligned} \quad (5)$$

where $T_1 = 1-\Gamma_1^C(\omega_R)-i\Gamma_1^S(\omega_R)$; $c_1^2 = (\lambda_{01}+2\mu_{01})/\rho,\quad c_2^2=\mu_{01}/\rho$.

The system of differential equations in partial derivatives (5) can be solved analytically, ie It obtained dispersion relations containing the Bessel function of the 1st kind of a complex argument. Dispersion relations are presented in the form of transcendental equation is solved by Muller. To do this, use the asymptotic Bessel functions for small and large values of the argument.

With the help of corresponding transformations [14] a system of differential equations (5) can be expressed as the beat

$$\frac{\partial^2\Delta}{\partial t^2}=c_1^2\nabla^2\Delta,\ \frac{\partial^2\omega_z}{\partial t^2}=c_2^2\nabla^2\omega_z,\ (6)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$.

A particular solution of equation (6) in the form:

$$\omega_z = W(r)e^{i(\omega t-n\theta)},\ \Delta = U_\Delta(r)e^{i(\omega t-n\theta)}\ . \quad (7)$$

Here W, U_Δ - Amplitude has a complex function that depends only of r. Wave number and phase velocity are expressed by the following formulas

$$\chi = \frac{2\pi}{\lambda}R; \quad c = \frac{\omega\lambda}{2\pi}$$

where χ - wave number, a is the phase velocity of the wave propagation.

To clarify their physical meaning, consider two cases:

1) $\chi = \chi_R$; $\omega = \omega_R + i\omega_I$ (or c = c_R +ic_I), then the solution (7) has the form of a sine wave in x, whose amplitude decays over time;

2) $\chi = \chi_R + i\chi_I$; $\omega = \omega_R$ (or c = c_R) then at each point x fluctuations established, but x decay.

Substituting (7) to (6) proceed to the next Bessel equations

Species:

$$\frac{d^2U_\Delta(r)}{dr^2} + \frac{1}{r}\frac{dU_\Delta(r)}{dr} + \left(\frac{\omega^2}{c_1^2T_1} - \frac{n^2}{r^2}\right)U_\Delta(r) = 0 \qquad (8)$$

The solution of equation (8), expressed in terms of cylindrical Bessel function of the 1st and 2nd kind of n-th order

$$U_\Delta(r) = \sum_{n=1}^{\infty}\left(A_nJ_n\left(\frac{\omega}{c_1T_1}r\right) + B_nY_n\left(\frac{\omega}{c_1T_1}r\right)\right), \qquad (9)$$

where J_n, Y_n -Bessel function of the 1st and 2nd kind of n - th order. In the disk center (r = 0) is assumed, that the movement is limited. With this expression conditions can be found where B = 0. Then, the solution takes the form:

$$\begin{pmatrix}\Delta \\ \omega_z\end{pmatrix} = \sum_{n=1}^{\infty}\begin{pmatrix} A_nJ_n\left(\frac{\omega}{c_1T_1}r\right) \\ D_nH_n\left(\frac{\omega}{c_1T_1}r\right)\end{pmatrix}e^{i(\omega t - n\theta)}; \qquad (10)$$

At the same time the movement of the cylindrical body (5), and (10) takes the form:

$$u_r = \sum_{n=0}^{\infty}\left(-A_n\frac{c_1^2T_1}{\omega^2}\frac{\partial J_n\left(\frac{\omega}{c_1T_1}r\right)}{\partial r} - i2D_n\frac{c_2^2T_1}{r\omega^2}J_p\left(\frac{\omega}{c_2T_1}r\right)\right)e^{i(\omega t - n\theta)}; \qquad (11)$$

$$u_\theta = \sum_{n=0}^{\infty}\left(A_n\frac{c_1^2T_1}{2\omega^2}J_n\left(\frac{\omega}{c_1T_1}r\right) - 2D_n\frac{c_2^2T_1}{\omega^2}\frac{\partial J_n\left(\frac{\omega}{c_2T_1}r\right)}{\partial r}\right)e^{i(\omega t - n\theta)}. \qquad (12)$$

Therefore, for (4), we obtain two set of boundary conditions which lead to two homogeneous equations with two unknowns A_n and D_n

$$A_n\left\{J_n(\chi\eta\frac{c}{c_2T_1})\left[0.5(c/c_2T_1)^2 - 1 + 1/\chi\right] - \frac{\eta(c/c_2T_1)}{\chi}J_{n+1}(\chi\eta\frac{c}{c_2T_1})\right\} +$$

$$+ i2D_n\left\{J_n(\chi\frac{c}{c_2T_1})\left[\eta^2(\frac{1}{\chi} - 1)\right] + \frac{\eta^2 c}{c_2T_1}J_{n+1}(\chi\frac{c}{c_2T_1})\right\} = 0,$$

$$iA_n\left\{\left[1-\frac{1}{\chi}\right]J_n(\chi\eta\frac{c}{c_2T_1})-\frac{\eta\ c}{c_2T_1}J_{n+1}(\chi\eta\frac{c}{c_2T_1})\right\}+$$

$$+2D_n\left\{\left[0.5(c/c_2T_1)^2-1+1/\chi\right]\eta^2J_n(\chi\frac{c}{c_2T_1})-\frac{\eta^2c}{c_2T_1\chi}J_{n+1}(\chi\frac{c}{c_2T_1})\right\}=0,$$

Where $\eta = c_2/c_1$.

To make such a system of equations have a nontrivial solution, the determinant of the coefficients must be zero. This condition gives the dependence of the frequencies (ω_R) and damping coefficients (ω_I) the wave number. Dispersion equation has the form:

$$\left\{\left[\frac{1}{2}\left(\frac{c}{c_2T_1}\right)^2-1+\frac{1}{\chi}\right]J_n\left(\chi\eta\frac{c}{c_2T_1}\right)-\left(\frac{\eta}{\chi}\right)\left(\frac{c}{c_2T_1}\right)J_{n+1}\left(\chi\eta\frac{c}{c_2T_1}\right)\right\}\cdot\left\{\left[\frac{1}{2}\left(\frac{c}{c_2T_1}\right)^2-1+\frac{1}{\chi}\right]J_n\left(\chi\frac{c}{c_2T_1}\right)-\left(\frac{1}{\chi}\right)\left(\frac{c}{c_2T_1}\right)J_{n+1}\left(\chi\frac{c}{c_2T_1}\right)\right\}- \quad (13)$$

$$-\left\{\left[\frac{1}{\chi}-1\right]J_n\left(\chi\frac{c}{c_2T_1}\right)+\left(\frac{c}{c_2T_1}\right)J_{n+1}\left(\chi\frac{c}{c_2T_1}\right)\right\}\left\{\left[\frac{1}{\chi}-1\right]J_n\left(\chi\eta\frac{c}{c_2T_1}\right)+\eta\left(\frac{c}{c_2T_1}\right)J_{n+1}\left(\chi\eta\frac{c}{c_2T_1}\right)\right\}=0.$$

Numerical results.

If you know the n = 0 and 1, we can calculate the Bessel and Neumann functions of any order of the following recurrence relations (F_n=J_n;Y_n)**:**

$$F_{n+1}(z)=\frac{2\pi}{z}F_n(z)-F_{n-1}(z),$$

where z – complex value.

$$V_0(\rho,\phi)=\sum_{k=o}^{\infty}(-1)^k\frac{\left(\frac{\rho}{2}\right)^{2k}}{(k!)^2}\sin 2k\phi=U_0(\rho,\phi). \quad (14)$$

Complex number z = x+iy may be represented as $z=\rho e^{i\phi};\rho=\sqrt{x^2+y^2},\ \phi=arctg\frac{y}{x}.$

and obtain:

$$J_0(\rho e^{i\phi})=\sum_{k=0}^{\infty}(-1)^k\frac{(\frac{\rho}{2})^{2k}}{(k!)^2}e^{2k\phi}=U_0(\rho,\phi)+iV_0(\rho,\phi)$$

$$U_0(\rho,\phi)=\sum_{k=o}^{\infty}(-1)^k\frac{\left(\frac{\rho}{2}\right)^{2k}}{(k!)^2}\cos 2k\phi=U_0(\rho,\phi)\quad .$$

Table 1. Some values of the Bessel functions in.Depending on arguments ($\phi=10^{\circ}$).

z	J_0 (z)		$Y_0(z)$	
0.0	0.99041	-0.00021	-1.97937	0.11159
0.1	0.99765	-0.00085	-1.53476	0.11269
0.2	0.99062	-0.00340	-1.08176	0.11597
0.3	0.97895	-0.00761`	-0.80837	0.11999

For series (14) is not greater than the remainder of the first discarded term. If you choose to $U_0(\rho, \phi)$ and $V_0(\rho, \phi)$ 26 members of the series (polynomials of 50th degree in ρ), the error will be smaller in absolute value $1.04\left(\frac{p}{2}\right)^{52}\frac{1}{(26!)^2}$, a maximum value (for ρ <10) approximately equal to $1.5 \cdot 10^{-17}$.

Results of calculations are shown in Table 1. As the relaxation nucleus viscoelastic material take a three-parameter core $R(t)=\frac{Ae^{-\beta t}}{t^{1-\alpha}}$ Rzhanitsina -Koltunova [7], having a strong singularity, where A, α, β - material parameters [7]. Assume the following parameters: $A = 0{,}048;\ \beta = 0{,}05;\ \alpha = 0{,}1$. The transcendental equation (13) are solved by Muller. At the same time, we have identified two root (13) (ν=0,3; $\chi = 100$) $c^{(1)}(100) = 0{,}919\,c_2$, $c^{(2)}(100) = 1{,}032c_2$. From sources [4,5,6] it is known that when $\chi = 98$, $c^{(1)}=0{,}92c_2$, and the Rayleigh wave velocity $c_R=0{,}9194c_2$. Results are presented in Figure 1. Note the characteristics of the curve 1, the phase velocity tends to infinity when the wave number is zero. A wave number tends to infinity, the phase velocity of the Rayleigh wave tends to speed for a half.

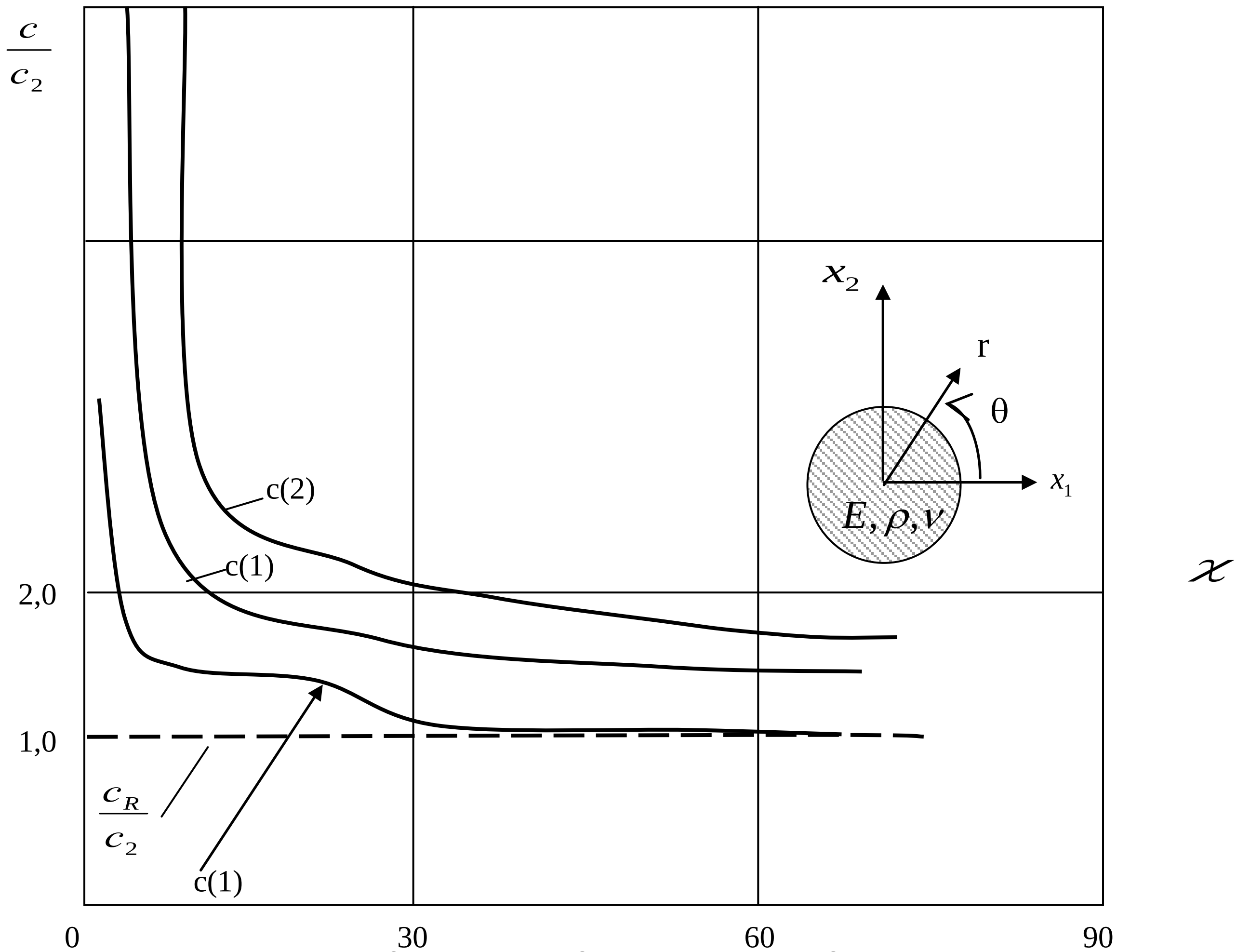

Fig. 1. The dependence of the real part of the phase velocity of the wave number.

The first and the second mode, the wave number tends to zero, has a cut-off frequency, ie, the phase velocity tends to infinity. At large wave numbers limit the phase velocity of the mode coincides with the velocity of the Rayleigh wave. At the cutoff frequency radial displacements are zero, and the cylinder is deformed flat in a static state. In the second mode to the cutoff frequency observed only real parts and imaginary parts adopt finite values with the desire of the wave number to zero.

In contrast to the well-known, in this case, in addition to the speed of Rayleigh waves with large wave numbers $C_R^{(1)}$ there exists a countable set of imaginary velocity (fig. 2)

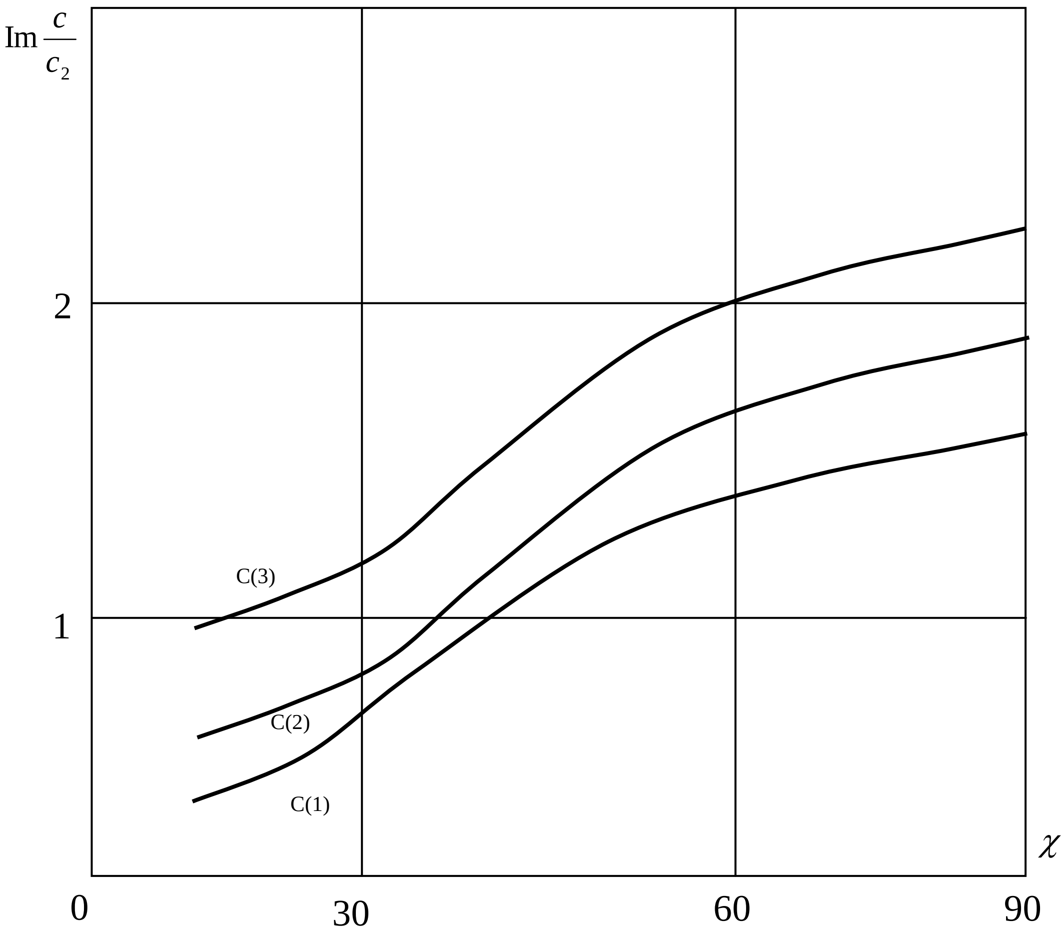

Fig.2. The dependence of the imaginary part of the phase velocity of the wave number.

On the surface of the cavity and apply the Rayleigh wave, but the complex (Fig 2).

$$\frac{A_n}{D_n}=\frac{-2i\eta^2\left\{J_n(\chi z)\left[\frac{1}{\chi}-1\right]+zJ_{n+1}(\chi z)\right\}}{\left\{J_n(\chi\eta z)\left[0.5z^2-1+\frac{1}{\chi}\right]-\frac{\eta}{\chi}zJ_{n+1}(\chi\eta z)\right\}}\ . \quad (15)$$

If we use (11), (12) and (15), we obtain the following amount of displacement

$$\frac{u_r}{D_nR_1}=\frac{2\pi}{z}\left\{\left[-\eta J_{n+1}(\chi\eta z\frac{r}{R_1})+\frac{R_1}{zr}J_n(\chi\eta z\frac{r}{R_1})\right]L_n-\frac{R_1}{\chi zr}J_n(\chi z\frac{r}{R_1})\right\}e^{i(\omega t-n\theta)}\ ,$$

$$\frac{u_\theta}{D_nR_1}=\frac{2}{z}\left\{\frac{R_1}{zr}L_nJ_n(\chi\eta z\frac{r}{R_1})+\frac{1}{\chi}J_{n+1}(\chi z\frac{r}{R_1})-\frac{R_1}{\chi zr}J_n(\chi z\frac{r}{R_1})\right\}e^{i(\omega t-n\theta)},\ (16)$$

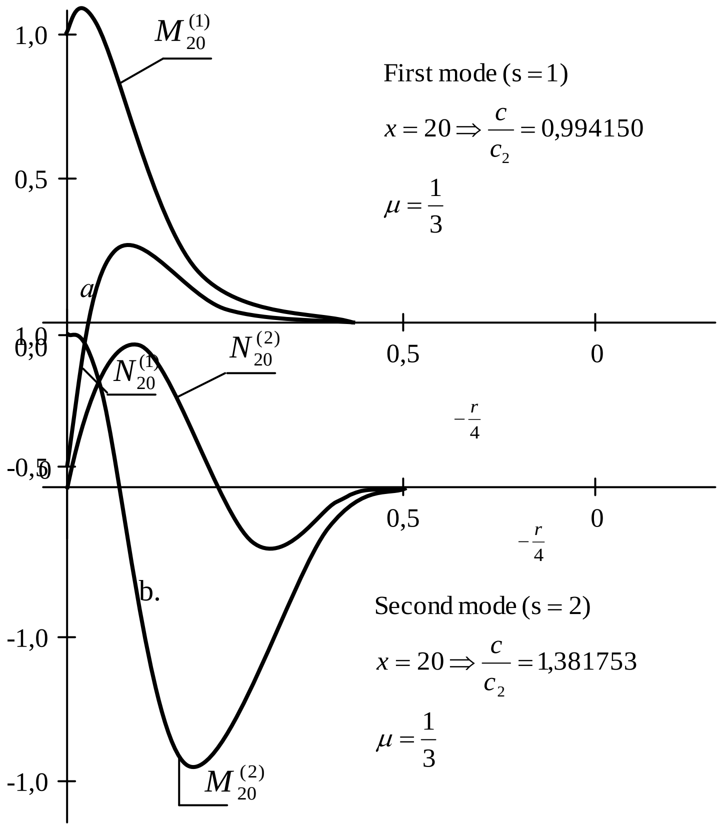

Figure 3. Respective first waveform (a) and second (b) phase velocities.

Where

$$z = c/c_2 T_1, \quad L_n = \frac{J_n(\chi z)\left[\frac{1}{\chi} - 1\right] + zJ_{n+1}(\chi z)}{\chi J_n(\chi\eta z)\left[\frac{1}{2}z^2 - 1 + \frac{1}{\chi}\right] - \eta z J_{n+1}(\chi\eta z)}.$$

General expression (16) we have

$$\frac{u_r}{D_n R_1} = Ampl(u_r)e^{i(\omega t - n\theta + \pi/2)}, \quad \frac{u_\theta}{D_n R_1} = Ampl(u_\theta)e^{i(\omega t - n\theta + \pi/2)}$$

The amplitudes of the movements represented in the form

$$M_n^{(s)} = \frac{Ampl(u_r)}{Ampl(\bar{u}_r)} = \frac{u_r}{(\bar{u}_r)_{r=R}} = \frac{L_n\left[-\eta J_{n+1}(\chi\eta z\frac{r}{R_1}) + \frac{R_1}{zr}J_n(\chi\eta z\frac{r}{R_1})\right] - \frac{J_n(\chi z\frac{r}{R_1})}{\chi z\frac{r}{R_1}}}{L_n\left[-\eta J_{n+1}(\chi\eta z) + \frac{1}{z}J_n(\chi\eta z)\right] - \frac{J_n(\chi z)}{\chi z}},$$

$$N_n^{(s)} = \frac{Ampl(u_\theta)}{Ampl(\bar{u}_r)} = \frac{L_n\frac{J_n(\chi\eta z\frac{r}{R_1})}{z\frac{r}{R_1}} + \frac{J_{n+1}(\chi z\frac{r}{R_1})}{\chi} - \frac{J_n(\chi z\frac{r}{R_1})}{\eta z\frac{r}{R_1}}}{L_n\left[-\eta J_{n+1}(\chi\eta z) + \frac{1}{z}J_n(\chi\eta z)\right] - \frac{J_n(\chi z)}{\chi z}}.$$

Numerical calculations are performed for $\nu = 0.33, \eta = 1/\sqrt{3}$. The calculated displacement amplitude shown in Fig. 3 and Figure 4. From the figures it is clear that movements are localized on the surface of the cylinder.

Conclusions.

1. It has been established that there is an infinite set of roots of the transcendental equation (13), the first root at large tends to Rayleigh-wave speed c=0,92c_2 . The phase velocity tends to infinity when the wave number is zero, ie, there is a cut-off frequency.
2. It was revealed that the cylindrical disc movement localized on the surface of the cylinder.
3. Accounting viscous properties of the material reduces the phase velocity values on 10 – 15%.

References

1.Uwhite. The surface elastic waves. -Trudy Institute of Electrical and Electronics Engineers. 1970, 58, №8, p.68-110.
2.Ewing W. M., Jardetzku W. S ., Press F .Elastic Waves in Layered Media, McGraw – Hill, New York, 1962.
3. R.M. Davis. Stress waves in solids. -Moscow. Foreign Literature Publishing House, 1961, -104p.
4. Safarov II Oscillations and waves in dissipative nedorodnyh environments and structures. Tashkent. sciences, 1992. -250p.
5. Safarov I..I, Akhmedov M.Sh, Boltaev Z.I. Oscillations and diffraction of waves on the cylindrical body in a viscoelastic medium. LambertAcademicPublishing (Germany) . 2016. 262p.
6. Safarov I..I, Akhmedov M.Sh, Boltaev Z.I.. Wave processes in mechanical waveguide. LAP LAMBERT Academic publishing (Germany). 2012.,217 p.
7. Koltunov M.A.. Creep and relaxation. M .: Higher School, 1976.-277 p.
8. Koltunov M.A., Mayboroda V.P., Zubchaninov V.G.. Strength calculations of products from polymeric materialov.- M: Mechanical Engineering, 1983.-239 p.
9. V.V. Bolotin, Yu. Beginners.. The mechanics of multilayered structures. -M .: Engineering, 1980.- 375 p.
10. Bazarov M.B., Safarov I.I., Shokin Y.M.. Numerical simulation of vibrations dissipative heterogeneous and homogeneous mechanical systems. -Novosibirsk, Siberian Branch of the Russian Academy of Sciences, 1996. -189 p.
11. Kayumov S., Safarov I.I.. Propagation and diffraction of waves in a dissipative - inhomogeneous cylindrical deformable mechanical systems. -Tashkent; sciences, 2004,215 p.
12. Safarov I..I, Akhmedov M.Sh, Boltaev Z.I.. Eigen values waves in layered media. Lambert Academic Publishing (Germany) . 2016. 192p.
13. Grinchenko V.T., V.V. Maleshko. Harmonic Waves in elastic telah.- K .: Thought Science, 1981, -283p.
14. SafarovI.I, BoltaevZ. I., AkhmedovM. Sh. Properties of wave motion in a fluid-filled cylindrical shell/ LAP, Lambert Academic Publishing . 2016 -105 p.
15.Safarov I.I, Akhmedov M. Sh ., Boltaev .Z.I. Natural oscillations and diffraction of waves on the cylindrical body. Lambert Academic Publishing (Germany) . 2016. 245p.